Ulud. Üniv. Zir. Fak. Derg., (1992) 9: 63-68

Applications of Singularity Functions For Designing Farm Machinery Shafts

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SUMMARY

The main purpose of this study is to demonstrate that the singularity function as an analytical method, beside the numerical analysis techniques like finite elements method, are easily applied to determine design parameters for power transmission shafts which are widely used in farm machineries. The step size is important to integrate singularity functions on computer. Because the T-Solver Plus software performs integration based on the iterations of the step functions, the shear force, moment and the deflection of the shaft at any chosen locations can be easily calculated due to the arbitrarily chosen step size. Depending upon the amount of loads on the shaft, calculated deflection values are used to find out the critical speed of the shaft.

Key Words: Singularity Functions, Shaft, Shear Force, Moment, Deflection, Critical Speed.

ÖZET

Tarım Makinaları Şaftlarının Dizaynlarında Tekil Fonksiyonlarının Uygulanmaları

Bu çalışmanın temel amacı, tarım makinalarında yaygın olarak kullanılan güç iletim millerinin dizayn parametrelerin saptanmasında sonlu

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elemanlar gibi sayısal analizlerin yanında, analitik bir metod olan tekil fonksiyonlarının kolaylıkla uygulanabilirliğini göstermektir. Tekil fonksiyonlarının bilgisayarda integrallerinin alınmasında adım büyüklüğünün önemi vardır. Tk-Solver Plus yazılım paketi bu işlemi, adım fonksiyonlarının, iteraksiyonuna bağlı olarak gerçekleştirdiğinden, seçilen adım büyüklüğüne göre şaftın herhangi bir noktasındaki kesme kuvveti, eğilme momenti ve sarkım kolaylıkla hesaplanabilir. Şaftın kritik devir sayısının bulunmasında milin yüklenme durumuna bağlı olarak hesaplanan sarkım değerlerinden yararlanılır.

Anahtar Sözcükler: Tekil Fonksiyonları, Şaft, Kesme Kuvveti, Moment, Sarkım, Kritik Hız.

INTRODUCTION

Power transmission shafts in farm equipments are generally supported at each end and are powered by gear-gear, chain-gear, or belt-pulley systems. Because of their supported two end points, they are considered as simply supported beams. Designing for farm machinery shafts requires numerical or analytical calculation techniques. The finite elements method has been widely used for designing power transmission shafts based on numerical analysis in recent years. On the other hand, an analytical solutions can be carrired out to calculate optimum design parameters of power transmission shafts.

MATERIAL AND METHODS

A simply supported and dynamically loaded transmission shaft generates shear force, bending moment, rotation and deflection throughout the beam. All these parameters can be described with mathematical functions using the strain energy model which is known Castigliano's theorem in literature (Shigley and Mitchell, 1983). The Castigliano's theorem is given as;

$$\delta_i = \frac{\partial U}{\partial F_i} \qquad (1)$$

where, δ_i is the deflection of the beam with respect to the applied force (F_i), U is strain energy which is the potential energy stored into a elastic member due to the deflection. The amount of the strain energy for a deflected simply supported beam is also depend on the dimension and modulus of elasticity of the shaft. Starting with the strain energy concept, the loading function of the shaft can be obtained as a singularity function which is sometime called load intensity (Deutschman, 1975). In this case the loads and the loading functions of power transmission members should be described precisely. Therefore, the solutions are in the form of concentrated moment, shear, step, ramp or parabolic singularity functions depending on the loads. Table 1 shows the common type of the singularity functions and their notations.

Table: 1
Notations and Meaning of Singularity Functions

Functions	Notation	Meaning	
Con. Moment	$<x-a>^{-2}$	if $x=a$ then $F(x)=1$	otherwise 0
Con. Shear	$<x-a>^{-1}$	if $x=a$ then $F(x)=1$	otherwise 0
Step	$<x-a>^{0}$	if $x < a$ then $F(x)=0$	otherwise 1
Ramp	$<x-a>^{1}$	if $x < a$ then $F(x)=0$	otherwise $x-a$
Parabolic	$<x-a>^{2}$	if $x < a$ then $F(x)=0$	otherwise $(x-a)^2$

The shear and the moment are always zero when $x \neq a$. Similarly, the step, the ramp and the parabolic functions generate zero only if $x < a$ according to the Table 1.

The power transmission shaft is cut with a imaginary plane for calculation moment and shear in order to have $x = x_1$ where the x_1 is the length of the segment. The analysis is carried out the right side of the shaft after the imaginary cut. The left handed side of the beam is usually ignored and the total force acting the beam is calculated by considering the equilibrium position at the cutting surface. Similarly, the total moment is the summation of the moment that acts on the shaft due to the total shear forces in equilibrium. From the results of these explanations for a given specific simply supported beam, the loading function is given as;

$$q = EI \frac{d^4y}{dx^4} = - F_1 <x-a>^{-1} - F_2 <x-b>^{-1} \tag{2}$$

where, F_1 and F_2 are loads on the beam. Using the load intensity function the shear and the moment equation can be obtained by integrating the load function one and two times over the segment length respectively. Therefore, the shear force and the moment equation as singularity functions are;

$$v = EI \frac{d^3y}{dx^3} = \int_{\infty}^{x} q \, dx = - F_1 <x-a>^{0} - F_2 <x-b>^{0} + c_1 \tag{3}$$

$$M = EI \frac{d^2y}{dx^2} = \int_{\infty}^{x}\int_{\infty}^{x} q\, dx = -F_1 <x-a>^1 - F_2 <x-b>^1 + c_1x + c_2 \qquad (4)$$

where, E is the modulus of elasticity, I is the moment of inertia, y is the deflection of the beam with respect to applied loads. Using initial and the boundary conditions, integration constants, c_1 and c_2, are calculated. In this system, initial and boundary conditions are; $M(0) = 0$, $M(l) = 0$, $y(0) = 0$, and $y(1) = 0$ respectively. Here, l denotes the length of the shaft. Boundary and initial conditions explain that there is no moment and deflections of the simply supported beam acting on supported points for this particular system. In Table 2., input parameters are given for a sample solid circular shaft in order to test calculations.

Table: 2
Input Parameters for a Power Transmission Shaft

Loads (N)		Location (cm)		Diameter	Length	M. Elasticity
F_1	F_2	F_1	F_2	d (cm)	l (cm)	E (N/cm^2)
30	120	20	70	3	100	$20*10^6$

If the deflections on the simply supported power transmission shaft are precisely calculated at the point of the loadings, the next step is to calculate the critical speed (ω) of the shaft. The critical speed is the measure for minimum revolution per minute of the shaft to have no high vibration effects. The critical speed is given in literature (Spotts, 1978; Okursoy, 1988) according to the Rayleigh-Ritz method which is;

$$\omega = \frac{1}{2\pi} \sqrt{\frac{g \sum_{i=0}^{n} y_i F_i}{\sum_{i=0}^{n} y_i^2 F_i}} \qquad (5)$$

where F_i's are applied forces, y_i's are deflections due to these forces, and g is the gravitational acceleration.

DISCUSSION AND RESULT

The shear and the moment diagrams as well as the loadings of a simply supported power transmission shaft are given in Figure 1. Singularity functions of the moment and the shear equations were solved on computer using Tk-Solver Plus programming package. The analysis were carried out in the application of the step functions for those equations. Calculations were able to performed in any step size. As can be seen from the Figure 1., the maximum shear, moment and deflections were calculated for a particular shaft as 90 N, 2700 N.cm, 0.029 cm respectively. The critical rotation is 1920 rpm that the shaft should be allowed as minimum rotation per minute for safety reasons.

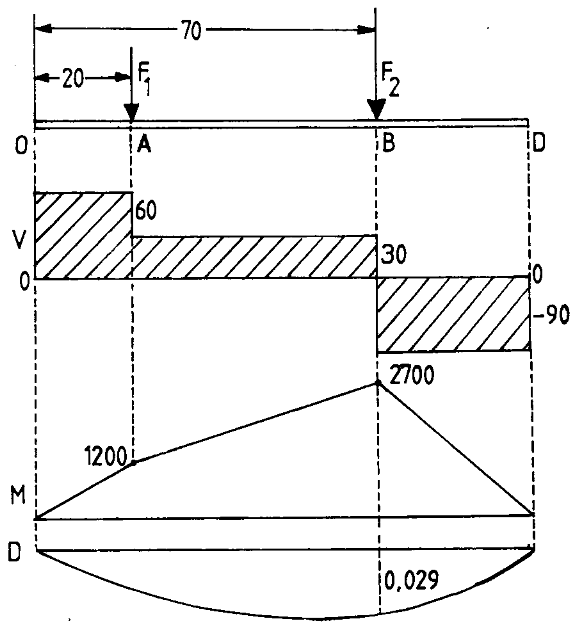

Figure: 1
The calculated shear, moment and the deflection diagrams of the sample shaft using singularity functions

As a result, like numerical techniques, singularity functions can be used for designing power transmission shafts. Singularity functions can be able to solved using Tk-Solver Plus programming package at any step size. Since the method are based on the computer iteration technique, increasing step size requires less calculations.

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